

# Crutch Tip for Swing-through Crutch Walking Control Based on a Kinetic Shape

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**Abstract**—This paper illustrates the dynamic effects of using a kinetic shape as a crutch tip on swing through crutch walking (non-weight bearing). The overground crutch walking of four participants was measured to examine the effect of a Kinetic Crutch Tip (KCT) on step length and swing time using a ProtoKinetics® Zeno Walkway System. Changes in ground reaction forces during the crutch gait cycle were examined by having the participants walk on an instrumented treadmill. We quantify changes in crutch dynamics by comparing results to standard rubber point tip crutch walking. The results showed that introducing a KCT to crutch walking can alter step length and swing time asymmetries during overground walking. Participants walking with a forward forcing KCT experienced a reduction in the horizontal ground reaction forces of up to 74% compared to walking on standard rubber crutch tips. The backward forcing KCT reduced the heel strike peak forces by as much as 27%. These findings show that crutch walking dynamics can be customized and optimized to yield a specific crutch walking behavior tailored to various user needs or walking environments.

## I. INTRODUCTION

The majority of current crutch design variations focus on the user interaction to increase comfort and decrease upper-limb support stress. Much less effort has been spent on finer control of crutch dynamics caused by crutch-ground interactions during use. While upper-limb support comfort is a significant aspect in crutch design, it is an incomplete picture without considering the behavior at the crutch-ground interface. By looking at this often overlooked interaction, we are examining the possibility of changing the crutch tip shape in order to systematically manipulate crutch walking dynamics. Having customizable crutch dynamics may make it possible to solve the issues of comfort and efficiency while aiding in the rehabilitation of crutch users. For instance, if a crutch user suffers from plantar faciitis [1] and needs to limit overexertion of poor bio-mechanics, a Kinetic Crutch Tip (KCT) may be configured to reduce impact forces experienced at heel strike. The force changes at the crutch-ground interface are directly related to stresses experienced by the user's wrist, elbow, and shoulder. Therefore, any ability to manipulate crutch-ground reaction forces would allow for the systematic reduction of these stresses.

To accomplish this optimization of forces, a kinetic shape [2] using a non-constant radius crutch tip can be used to systematically change the user's dynamics. Since a non-constant radius wheel can redirect an applied user's weight into a rolling force [3], a similar shape applied to the tip

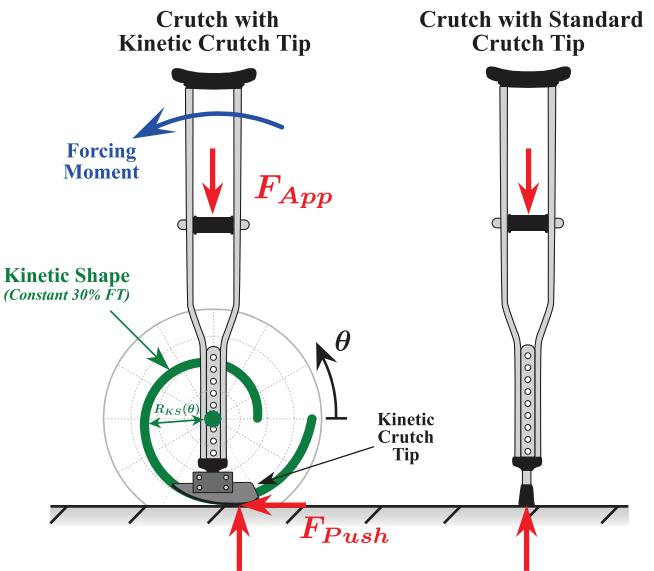


Fig. 1: (*Left*) A crutch with a kinetic crutch tip is able to predictably redirect applied forces ( $F_{App}$ ) into the forward or backward direction depending on its kinetic shape definition. (*Right*) A standard crutch with a regular small constant radius tip only reacts perpendicularly to the ground, while offering no horizontal rolling force ( $F_{Push}$ ).

of a crutch can generate different crutch walking dynamics compared to a standard point tip. The difference in system forces between a kinetic shape and a standard crutch tip can be seen in Figure 1. This formulation of a shape allows us to methodically form the crutch tip such that it will yield a certain type of user dynamics (i.e., forces and motions).

Our hypothesis is that using a KCT will allow for the systematic adjustment of crutch walking dynamics to suit user-specific needs. We analyzed temporal and spatial stepping parameters, while also examining ground reaction forces in order to quantify the effects of a KCT as compared to a standard crutch tip.

## II. BACKGROUND

Throughout history, the walking crutch has been used as a type of walking assistance device. A form of crutches categorized as rigid supporting staffs with an underarm cross piece can be traced back to the ancient Egyptian ages [4].

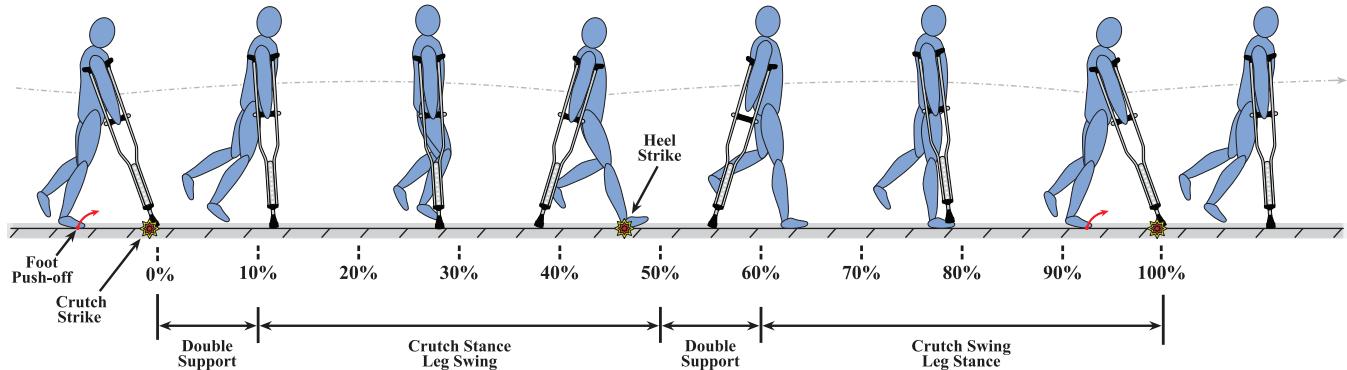


Fig. 2: Swing-through non-weight bearing crutch gait cycle.

Later adaptations appeared in 1917 from Emile Schlick for a crutch-like device, which was structurally very much like its predecessors [5]. Although its skeletal material, comfort padding, and crutch tip have been optimized over time, the fundamental design of this crutch is still widely used today and known as an axillary crutch, shown in Figure 3-C1. Axillary crutches, which are commonly used to assist in acute walking disabilities, are so named because their familiar padded tops are placed into the axilla (underarm). The user's body weight is transferred to the hand-grip directly and the axilla indirectly.

This general crutch design concept continues today through most crutch-like assistive devices, where the user supports themselves and swings over the crutch. This type of crutch gait cycle is known as swing-through non-weight bearing crutch walking and is shown in Figure 2. During this type of crutch walking, as the crutch user lands and pivots over a crutch, repeated and acute stresses of crutch hand/arm support are channeled through the user's wrist, elbow, and shoulder joints. These repetitive stresses can leave chronic crutch related injuries such as stress fracture of the ulna [6], compression of the radial nerve [7], and

axillary artery injury [8]. Crutch walking movements alter the foot heel strike and push-off (plantar flexion) forces, adding stress to the user's foot [9]. Further, the effort of swing-through crutch gait has a higher net metabolic cost per unit distance than running [10], leaving users fatigued, while limiting their everyday crutch walking range. As the user pivots over the crutch during crutch stance, they limit their movement control exposing themselves to potentially devastating falls. Although the daily number of users is estimated to be 800,000 in the US alone [11], chronic stress-induced injuries, physical demand, fear of falling, and plain frustration make crutches one of the most frequently abandoned assistive devices [11].

Crutch technology advances have partially alleviated these crutch walking problems. One type is the forearm (a.k.a. elbow, lofstrand, Canadian) crutch (Figure 3-C2), which lets the user support their body weight on their forearm. This eliminates the risk of the user applying repeated high pressures to their axilla, thus reducing the risk of crutch paralysis [7]. The Millennial® In-Motion spring-loaded crutch (Figure 3-C3) uses a compression extension spring to soften crutch impact and slightly restore crutch walking energy lost during crutch strike. The SureFoot® Strutters is a spring-assisted four-bar linkage type of crutch, which offers extra user stability through a larger support base and upper torso comfort (Figure 3-C4). In comparison to standard axillary crutches, the Strutters shows a mean peak palmar force reduction of 45% [12]. A notable popular crutch design are the Mobilegs® crutches (Figure 3-C5), which create user comfort by extending axilla support area and so reducing supporting pressure.

Although there have been improvements in crutch design, improvements generally target crutch-user interaction such as crutch grip and torso support. Limited research has been done to advance crutch-ground interactions in order to modify or control user dynamics. The available crutch tips shown in Figure 3 T1–T5 have a rubber point tip or a slightly compliant point tip, always mimicking a constant radius (i.e., no radius change) when rolled over. A point or constant radius tip cannot assist or resist the user during swinging or rolling over the crutch tip, and all forward progression forces are generated by the user pushing themselves forward over the crutch and into heel strike. Crutches do exist with a larger radial crutch tip, however these crutches have a large constant radius that also does not change [13] [14].

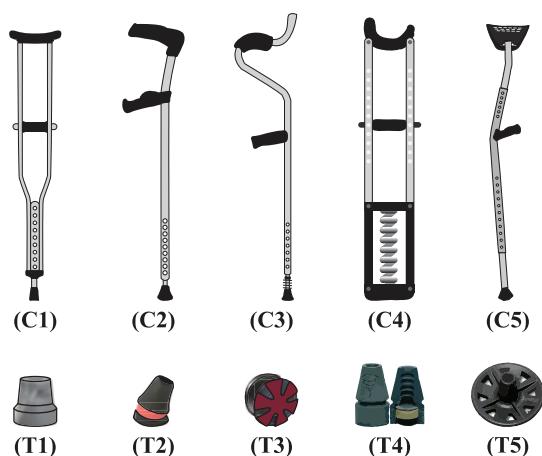


Fig. 3: (C1) Axillary Crutch, (C2) Forearm/Elbow Crutch, (C3) Millennial® In-Motion Spring-Loaded Crutch, (C4) SureFoot® Strutters, (C5) Mobilegs® Crutch, (T1) Standard Solid Rubber Tip, (T2) Adventure® Pivoting Tip, (T3) NonSlip Tip, (T4) Tornado Gel Tip, (T5) SandPad® Tip

Crutch users include chronically disabled individuals that rely on their crutches for everyday ambulation. This group includes disabilities such as lower limb amputation, spinal bifida, cerebral palsy, muscular dystrophy, spinal cord injury, post-polio syndrome, osteoarthritis, or multiple sclerosis. Crutch users also include temporarily-injured individuals with acute conditions such as foot and leg fractures, tendon tears, hip and knee replacements, or other lower extremity injuries. While temporary crutch users may heavily rely on crutches for standard non-weight bearing mobility, partial weight bearing (PWB) mobility may also be achieved. It has been known that systematic PWB during the recovery period following foot and leg injuries or surgeries increases rehabilitative effects, speeding up the rate of recovery, and decreasing recurring injury [15] [16]. Compared to walkers and canes, crutch users have been found to achieve a target PWB on a limb twice as accurately, making them a highly effective tool for PWB therapy [16]. It would be very desirable to be able to manipulate the crutch-ground forces during crutch stance and in turn the crutch user's dynamics such that it produces movements and forces that correspond to a specifically targeted mode of ambulation and PWB. Thys et al. [17] states: "*If the dynamic behavior of crutches could be improved, it would greatly enhance their utility and could lead to improved rehab for locomotion deficient persons.*" This study investigates how the interaction between the crutch tip and ground affects the behavior of crutch walking dynamics.

### III. KINETIC SHAPE CRUTCH TIP DESIGN AND FABRICATION

The formulation of the kinetic shape allows a physical curve to be generated that produces a known over-ground horizontal rolling force as a vertical force is applied [2]. Essentially, it uses the idea that irregularly shaped objects tend to roll towards a shrinking radius just as a circular shape would roll down an incline. This concept has been used on a gait correcting shoe [3] [18], a musical instrument [19], but can generally be used in any mechanical situation requiring a known and exact force application that is dependent on a position. The two-dimensional kinetic shape equation in polar coordinates is described in (1); see [2] for the derivation.

$$R(\theta) = R(\theta_i) \exp \left[ \int \frac{F_r(\theta)}{F_v(\theta)} d\theta \right] \quad F_r(\theta) < F_v(\theta) \quad (1)$$

where  $\theta$  is the angular position around the kinetic shape origin,  $R(\theta_i)$  is the initial radius of the kinetic shape,  $F_r(\theta)$  is the desired horizontal ground reaction force function around the shape, and  $F_v(\theta)$  is the force function vertically applied onto the center of the shape ( $R(\theta)=0$  and perpendicular to the ground). Note that the behavior of the kinetic shape stays invariant as the shape is scaled up or down and is only dependent on the force ratio  $F_r(\theta)/F_v(\theta)$ .

Placing a properly defined kinetic shape on the bottom of a crutch can generate a customizable rolling effect based on the vertical force applied by the user's weight,  $F_v(\theta)$ , and the customizable horizontal force,  $F_r(\theta)$ , yielding the definition of the kinetic shape crutch tip,  $R(\theta)$ .

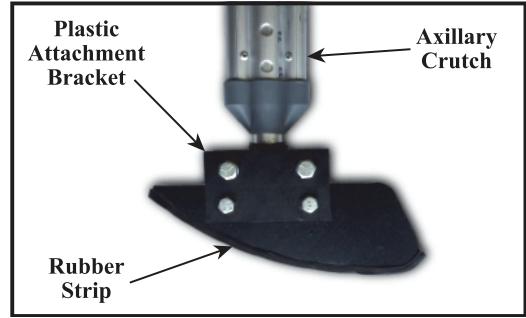


Fig. 4: Actual KCT assembly used in this study.

After a number of iterations and several preliminary qualitative analyses with kinetic shapes as crutch tips, we selected a constant 0.30 force transfer ratio ( $F_r(\theta)/F_v(\theta)$ ). Force ratios less than 0.3 did not have significant dynamic effects.  $R(\theta_i)$  was selected to be 20.0 cm (7.9 in). Only a sector angle range of the kinetic shape was used, which ranged from 4.3 rad (245°) to 5.2 rad (300°). The complete kinetic shape equation for our KCT is shown in (2) and can be seen in Figure 1.

$$R(\theta) = \pm 20 \exp(0.30\theta) \quad 4.3 \leq \theta \leq 5.2 \quad (2)$$

Note, that the sign of (2) is distributed from the  $F_r(\theta)$  term in (1), where a positive sign is produced by a forward forcing horizontal force, while a negative sign is produced by a backward forcing horizontal force. However, only one KCT was fabricated; turning the tip around 180° generates the opposite direction horizontal force. Our comparison used both the forward and backward configuration as well as a standard crutch tip.

The KCT, shown in Figure 4, was laser cut from a 1 cm (0.375 in) thick sheet of Acetal Resin (Delrin®) plastic using a 60 W laser cutter (Universal Laser System® VLS4.60). A 0.6 cm (0.25 in) thick strip of rubber (60 A Durometers) was screwed onto the rolling perimeter surface, where the attachment screws were countersunk into the rubber. The KCT was firmly fastened onto the bottom of an axillary crutch with a custom Acetal Resin (Delrin®) plastic bracket. The entire crutch tip assembly (shape and bracket) had a total weight of approximately 470 g (1.0 lb).

### IV. EXPERIMENTAL PROTOCOL

The experiments compared the dynamic effects of crutch walking when using different KCT and normal crutch tips. The experiment was split into two phases. Phase one focused on step length and swing time gait parameters, while also determining the steady state crutch walking velocity for each participant. In phase two, we measured the subjects' ground reaction forces over the entire crutch gait cycle.

Axillary crutches were used for all phases and trials. We adjusted crutch height and grip location according to crutch sizing standards for each participant. In order to compensate for the added weight of the KCT assembly, we attached matching lead weights to the crutches when using a standard tip. Each subject walked one trial per tip setting where the order of crutch tip setting was randomized for each participant.

### A. Subjects

Four healthy male subjects, ages  $24.25 \pm 1.7$ , with minimal to no crutch experience were included in this study. All subjects did not have any inherent gait or lower limb gait asymmetries and all wore non-constricting clothing with comfortable athletic shoes. Written informed consent was obtained from each subject prior to participation with a protocol approved by the Western Institutional Review Board.

### B. Phase 1: Overground Crutch Walking

We measured stride velocity, step length, and swing time for each participant using the ProtoKinetics® Zeno Walkway System (ProtoKinetics, LLC, Havertown, PA), which is a 2.0 ft (0.6 m) by 16.0 ft (4.9 m) walkway consisting of pressure sensors that are able to accurately monitor each step position. We define foot step length as the distance between the point where the crutches first touch down to the location where the foot first touch down. Crutch step length is defined the same way, but between the feet and crutch locations. Swing time is the time interval during which either the foot or crutch are off the ground during a step.

Each participant was instructed to crutch walk for five minutes at a self-selected velocity over the Zeno Walkway. The participants walked back and forth over the Zeno Walkway using a normal tip, a forward forcing KCT, and a backward forcing KCT. Participants turned ( $180^\circ$ ) at a distance of approximately two strides before and after the mat to ensure steady state walking measurements. When turning, participants turned an approximately 0.5 m radius half-circle. Before each trial, the participant rested until their resting heart rate was achieved. During each five minute trial, the participant's continuous heart rate was recorded using a Bluetooth 4.0 Wahoo® TICKR heart rate monitor controlled

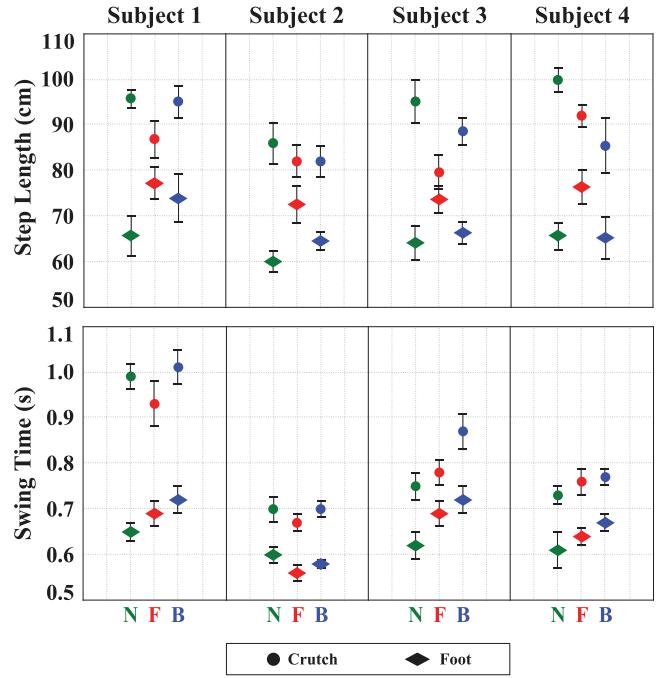


Fig. 6: Step length (top) and swing time (bottom) for overground crutch walking changed for each subject as a result of crutch walking with different crutch tips (i.e., Normal, Forward, and Backward).

with a custom mobile application. To correlate the gait data to when the participant reached a steady-state heart rate, we used a least square curve fit in MATLAB® with a step input dynamic response model defined in (3). This heart rate modeling method has also been applied by Su et al. [20].

$$HR(t) = HR_{SS} + (HR_{Rest} - HR_{SS})e^{-t/\tau} \quad (3)$$

where  $HR_{Rest}$  is the measured resting heart rate,  $HR_{SS}$  is the steady state heart rate, and  $\tau$  is the time constant. We defined the steady state heart rate as the heart rate after two time constants,  $2\tau$ , as shown in Figure 5. We determined the comfortable gait velocity for each participant as the average stride velocity during this steady state time interval averaged over the three trials. This is the velocity that was used for phase two treadmill velocity. Step length results can be seen in Figure 6.

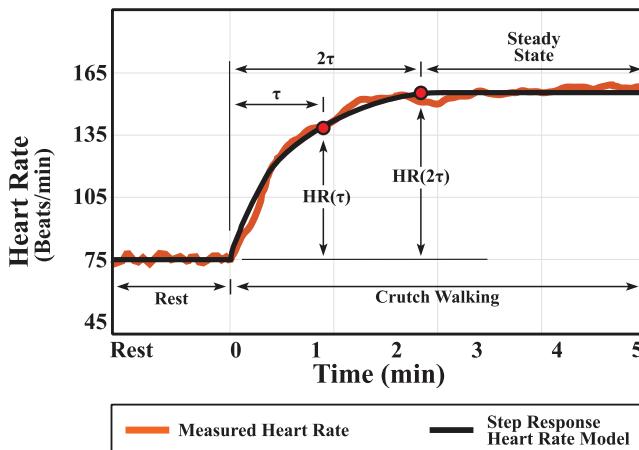


Fig. 5: In phase 1 the heart rate of each subject was recorded as they walked five minutes with a each crutch tip setting. The steady state crutch walking period was estimated by fitting a step response dynamic model onto the recorded heart rate. The average velocity in the steady state period of the fitted model of the normal crutch tip for each subject was used as the treadmill velocity in phase 2.

### C. Phase 2: Treadmill Crutch Walking

The participants were instructed to walk on a level instrumented split-belt treadmill with force plates (FIT, Bertec Corp., Columbus Ohio) underneath the treads which is part of the CAREN (Computer Assisted Rehabilitation Environment) system (Motek Medical, Amsterdam). Participants followed their same crutch tip trial pattern while walking for two minutes per trial. Treadmill velocity was set at the steady state velocity found in phase one. The instrumented treadmill measured horizontal (anterior-posterior/front-back) and vertical ground reaction forces of the participants during crutch walking at 100 Hz.

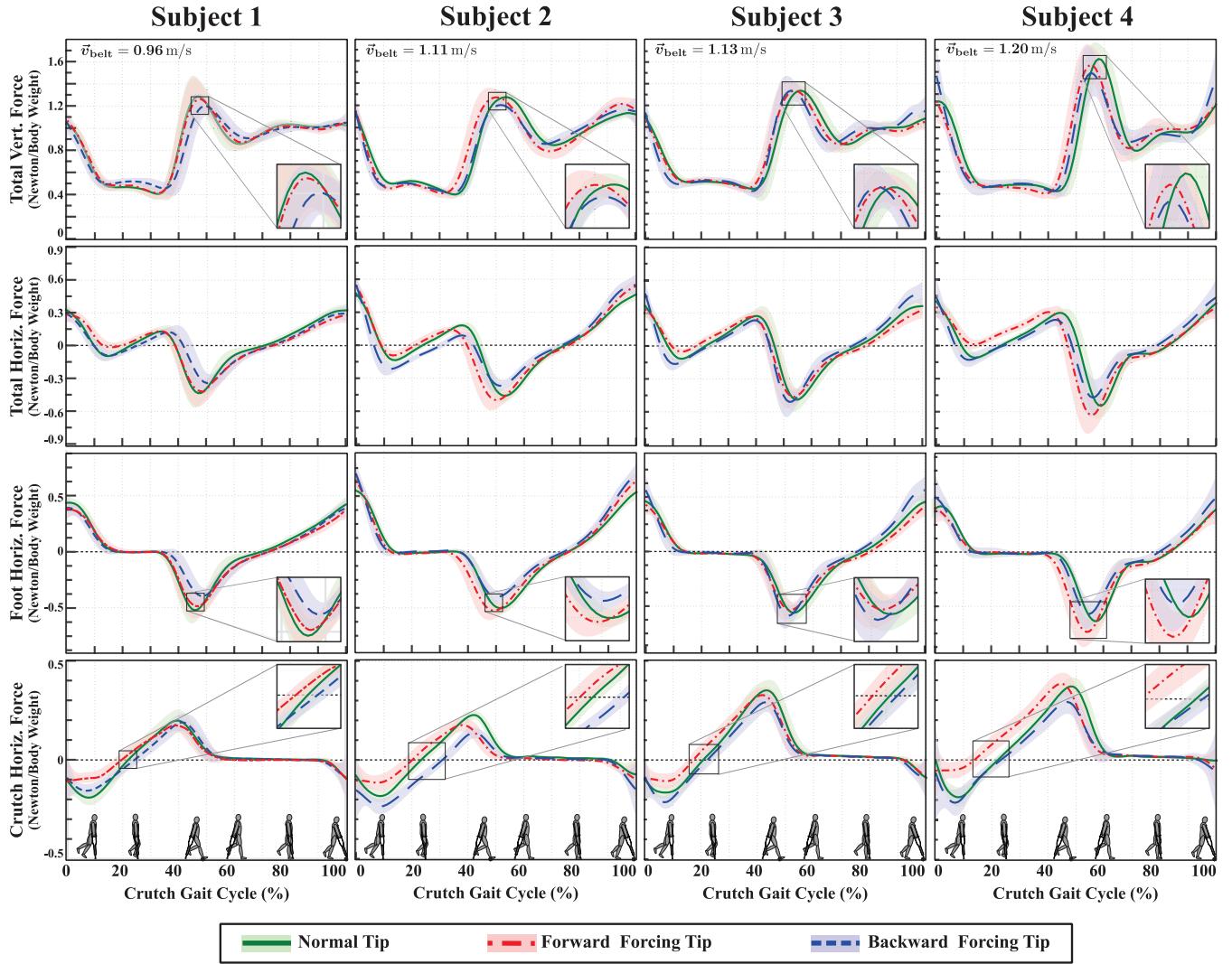


Fig. 7: Ground reaction forces measured during Phase 2, plotted as a function of the gait cycle (GC). It can be shown that crutch walking on kinetic shape crutch tips consistently altered ground reaction force magnitudes and timing.

## V. RESULTS AND DISCUSSION

The introduction of the kinetic shape to a crutch tip has a quantifiable effect on the dynamics of crutch walking. Figure 6 shows key trends in how the KCT changes participants' crutch and foot step length and swing time. The KCT reduces the difference between crutch and foot step length in both the forward and backward forcing orientations when compared to a normal constant-radius tip. Using the forward forcing KCT orientation, the foot step length is increased, while crutch step length is decreased for all subjects when compared to the normal tip. For three of the four participants, there is a clear trend in increasing foot swing time from normal to forward, and again from forward to backward KCT orientations. All participants showed the longest crutch swing time for the backward KCT. The difference between the anterior-posterior horizontal forces created by the forward and backward KCT results in a change in momentum and swing velocities of the user. This may lead to the observed crutch and foot swing time shown in Figure 6. Noticeable trends were observed in the ground

reaction forces when using the different KCTs (Figure 7). Equation 4 is used for quantitative comparisons between the scalar key values discussed.

$$\% \text{ Change} = \frac{K_{\text{Tip}} - N_{\text{Tip}}}{N_{\text{Tip}}} \cdot 100 \quad (4)$$

We denote the measured parameter value from the KCT (either forward or backward) as  $K_{\text{Tip}}$  and the measured parameter value from the normal crutch tip as  $N_{\text{Tip}}$ .

During crutch strike (0-20% GC), the forward forcing KCT reduced the posterior force by up to 74% from the normal tip, while the backward orientation increased it by up to 34% from the normal tip. It was observed that the ground reaction forces switch from posterior to anterior (equilibrium point) during crutch stance (Figure 2) around  $21 \pm 1\%$  GC for the normal KCT,  $17 \pm 3\%$  GC when using the forward KCT, and  $24 \pm 3\%$  GC for the backward KCT. This may be due to the shifting of the entire horizontal ground reaction force curve up for the forward KCT and down for the backward KCT. Among all subjects, the forward forcing KCT seems

to create a larger positive shift in horizontal ground reaction forces as the crutch walking velocity increases. Along with the force magnitudes during crutch stance, this time shift of crutch stance equilibrium causes changes in impulse ( $\int F dt$ ) during crutch strike and crutch push-off. The observed reduction in peak forces and impulses during crutch stance is predicted to alleviate stresses in the user's wrist, elbow, and shoulders [6], however joint forces were not directly measured in our study. During foot heel strike (40-60% GC) the forward forcing KCT increased the peak force by 15%, while the backward tip decreased the peak force by 24% both compared to the normal tip. Among all participants, the vertical heel impact force with a KCT was either equivalent or less than the impact force with a normal tip, however the crutch walking velocity between subjects appears to affect this peak force change.

There was no significant force profile difference between all tested KCTs during mid foot stance (60-85% GC). For three out of four participants, the horizontal foot push-off force (85-100% GC) was increased when using the backward KCT, indicating a slightly higher plantarflexion effort by the user to initiate crutch stance. Although the forward forcing KCT resulted in high crutch stance force profile modification, we did not observe any significant changes or trends using this crutch tip during foot stance or push-off.

Given these reported swing-through crutch dynamics, we expect that a kinetic shape applied as a crutch tip can be configured to generate specific and desirable movement and force profiles during crutch walking.

## VI. CONCLUSION

We examined the temporal, spatial, and kinetic effects of KCTs on swing through non-weight bearing crutch walking. These effects were compared to the dynamic characteristics of normal tip crutch walking. Both KCT orientations significantly reduced step length. Step time was also altered, however no discernible trend was observed. It was evident that we were able to manipulate crutch walking gait cycle ground reaction forces using a KCT. In all subjects, the forward forcing KCT created additional assistive forces during crutch stance, while a backward forcing KCT caused an increase in resistive forces. The changes in forces during crutch stance affected the subsequent leg stance phase forces. Horizontal and vertical heel strike ground reaction forces were reduced for three out of four subjects using a backward forcing KCT, while user foot push-off force increased for three out of four subjects.

These results indicate that a KCT may be configured to create desirable variations in crutch walking dynamics. We plan to further investigate KCTs with gait cycle specific curvatures in mind. Although these initial results are promising for the control of crutch dynamics, further work is needed to examine crutch walking energy expenditure and trajectories using a KCT. Since we were able to manipulate assistive and resistive crutch ground reaction forces on a flat surface, we hypothesize that a KCT is able to provide controlled resistance for downhill walking, while increasing assistance in uphill ambulation.

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