Proceedings of ASME's International Mechanical Engineering Congress and Exposition
IMECE 2017
November 3-9, 2017, Tampa, Florida, USA

IMECE2017-72068

ANALYSIS OF TWO-DIMENSIONAL KINETIC SHAPE SYSTEMS

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ABSTRACT

A Kinetic Shape has a physical and continuous curve with a changing radius that is exactly defined by its kinetic behavior. A Kinetic Shape curve is defined by specifying the force applied to the Kinetic Shape and the force with which the Kinetic Shape subsequently reacts at ground contact. This concept allows for predictable, position-dependent, and purely mechanical force redirection which make it broadly applicable. Kinetic Shapes have been previously used in several applications to predict the redirection of forces applied to the shape into ground reaction forces. Here, we analyze various ways 2D Kinetic Shapes interact and show how different mechanical force-based computational operations can be performed using these interconnected Kinetic Shapes, which we call Kinetic Shape Systems.

NOMENCLATURE

θ	Kinetic shape rotational position
$R(\theta)$	Physical kinetic shape curve
$F_{\perp}(\theta)$	Force applied on kinetic shape, perpendicular to ground
$F_{\parallel}(\theta)$	Ground reaction force, parallel to ground
F_{OUT}	Kinetic shape system output

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INTRODUCTION

A circle on a level surface will not move because there are no external forces, and it cannot reach a lower potential energy state. However, if there is asymmetry between the point of contact and the center of mass, an imbalance will occur, which will create a moment. A shape that changes radius in a certain way will convert a force perpendicular to the surface into a force parallel to the surface. A shape with such a continuous curve is named a Kinetic Shape (KS) [1, 2].

The concept of a KS is extended in this paper to include two or more KSs that are interconnected. In this case, the output force of one KS becomes the input to the next KS. We call these interconnected KSs a Kinetic Shape System (KSS). The KSS concept can be extended to N^{th}-order systems, but we focus our attention on second order systems in this paper to introduce how they can be defined and used.

BACKGROUND

When a two dimensional wheel is pressed onto a declined slope, the wheel will tend to roll down the slope. Likewise, when held at its polar center axis, a two dimensional and physical irregular shaped smooth curve that is pushed onto a level surface will tend to roll towards the curve's decreasing radius. This

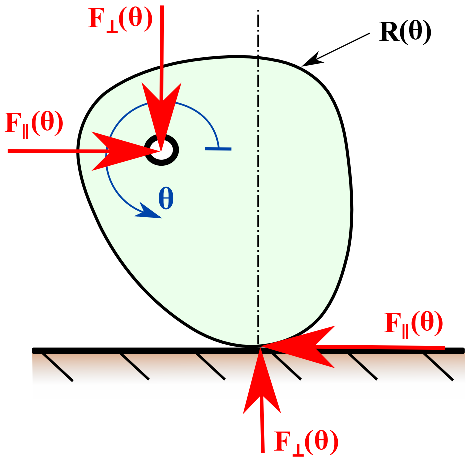

FIGURE 1. Kinetic and geometric definition of a kinetic shape (KS), the building block for kinetic shape systems (KSS). A physical curve $R(\theta)$ is defined by specifiying applied force $F_{\perp}(\theta)$ (perpendicular to ground) and reaction force $F_{\|}(\theta)$ (parallel to ground and tangent to shape curve)

is the general premise of the kinetic shape (KS) as described by Handzic et al. [1]. KSs are physical irregular curved (non-constant radius) shapes that are exactly defined by the forces applied to the shape and the reaction forces produced by the shape. A two-dimensional KS is shown in Figure 1 and defined in Equation 1.

$$R(\theta) = R(\theta_i)\ exp\left[\int \frac{F_{\|}(\theta)}{F_{\perp}(\theta)} d\theta\right] \quad (1)$$

$R(\theta)$ is the curve of the KS in polar coordinates, $R_i(\theta)$ is the initial radius of the KS, $F_{\perp}(\theta)$ is the force applied to the KS and perpendicular to the rolling surface, and $F_{\|}(\theta)$ is the ground reaction force parallel to the rolling surface and tangent to the KS curve. Note that the ground reaction force parallel to the ground is dependent on the KS curve and the applied force to the KS. Although KSs can be defined over more than one revolution $(0-2\pi)$, the physical design considerations for such a shape become more challenging.

This general concept allows for a wide range of applications where exact and predictable position-dependent mechanical force redirection is required. One example for the potential application of the KS can be found in rock climbing. The spring loaded camming device (SLCD) is used to secure a rock climber to a rock wall by inserting a set of opposing spiral cams into a crack [3]. As the rock climber falls or applies a sudden motion to the device, the cams expand inside the crack securing the climber to the wall. Essentially the cams redirect the climbers falling force outward onto the crack wall.

Another study combines the two-dimensional KS equation with the equation of a vibrating taut string to produce a novel variable tension string instrument [4]. In this instrument, a KS is attached to a string and as the KS rotates the plucked string produces predictable musical notes.

One study examines a deformable soft robot that is able to roll by manipulating its outside rim to attain a nonconstant radius (irregular shape) [5]. While the robot can be programmed to systematically move all planar directions, the study's authors do not present an analytical explanation to the robot's rolling kinetics. The KS concept can be used to analyze and predict this robot's motion.

Foot roll over shapes (ROS) are foot rocker shapes/curves that the foot rolls over when completing the stance phase during the walking cycle [6, 7]. Manipulation of ROS through footware and prosthetic design have been shown to greatly affect human walking motions. Although extensively researched, current biomechanical studies have not been able to analytically predict the behavior of ROS. The KS concept can assist in the design and analysis of ROSs.

The KS concept has been applied to rehabilitation and assertive devices. The Gait Enhancing Mobile Shoe (GEMS) is used by stroke patients to even their walking asymmetry [8, 9]. Its motions relate to that of a split belt treadmill. The GEMS uses KSs as its wheels in order to apply a predictable backward motion profile to the user's foot. The Kinetic Crutch Tip (Moterum MTip^{TM}) is a curved crutch tip where its curve is defined by the KS equation [10]. This allows this crutch tip to produce assertive and resistive ground reaction forces, optimizing the crutch walking cycle.

Naturally once the restraining force, $F_{\|}(\theta)$, is removed from the rotation axle, the KS will roll freely, however its rolling motion will be contingent on its curve and applied force. This motion of a unrestrained two-dimensional KS was studied by Handzic et al. [2]. Unrestrained dynamic KS can be used in situations where predictable position-dependent movement is needed.

It is easy to see the resemblance between a KS and other objects with eccentric rotation points, such as conventional cam [11]. Although the study of camming is generally focused on kinematics and tribology, it does not concentrate on the controlled forced redirection of continuous and smooth irregular shapes. KSs are not cams, however cams can be defined as KSs in order to predict kinetic aspects of the cam.

In this paper we will examine the systematic interaction of statically restrained KSs and will not consider their dynamic and relative movements once unrestrained. The performance of a interactive system of KSs has not been studied. Thus, this paper focuses on presenting the analytical and experimental analysis of situations where KSs interact and exchange forces.

KINETIC SHAPE SYSTEM (KSS) DEFINITION

While a single kinetic shape (KS) is able to predictably redirect a mechanical force in one stage, we are interested in examining the effects of multiple KSs interacting. We will name this kinetic connection between multiple KSs a Kinetic Shape System (KSS).

In its simplest form, a KSS consists of two KSs where the force output of one KS is applied to the second KS. The force profile output of such a KSS is dependent on the curve ($R(\theta)$), force profile definition ($F_{\parallel}(\theta)$, $F_{\perp}(\theta)$), and rotational position (θ) of both KSs. This fundamental KSS example can be seen in Figure 2. Note that the interaction of two KSs can be in the form of pushing or pulling.

Hereafter, a KS that is exerting a force onto another KS will be referred to as a *donor* KS, while a KS that is accepting forces will be referred to as a *receiver* KS. Further, the number of *degrees* of a KSS is the number of KSs involved in a KSS.

Any consecutive interaction between two KSs in a KSS can be defined by Equations 2 and 3.

$$R_1(\theta_1) = R(\theta_{i1})\ exp\left[\int \frac{F_{\parallel 1}(\theta_1)}{F_{\perp 1}(\theta_1)} d\theta_1\right] \tag{2}$$

$$R_2(\theta_2) = R(\theta_{i2})\ exp\left[\int \frac{F_{\parallel 2}(\theta_2)}{F_{\parallel 1}(\theta_1)}\ d\theta_2\right] \tag{3}$$

Here the subscripts 1 and 2 refer to the donor and receiver KS, respectively. Note that force profile, $F_{\parallel 1}$, appears in both equations. These relations include five variables, while including one shared variable. Depending on the application, three variables can be determined in advance and the others will be derived from these equations. For example, if the KS curves ($R_1(\theta)$,$R_2(\theta)$), KS rotational positions (θ_1, θ_2), force entering the donor KS ($F_{\perp 1}$), and force exiting the receiver KS ($F_{\parallel 2}$) can be measured, it is possible to solve for the joining force ($F_{\parallel 1}$) between the KSs.

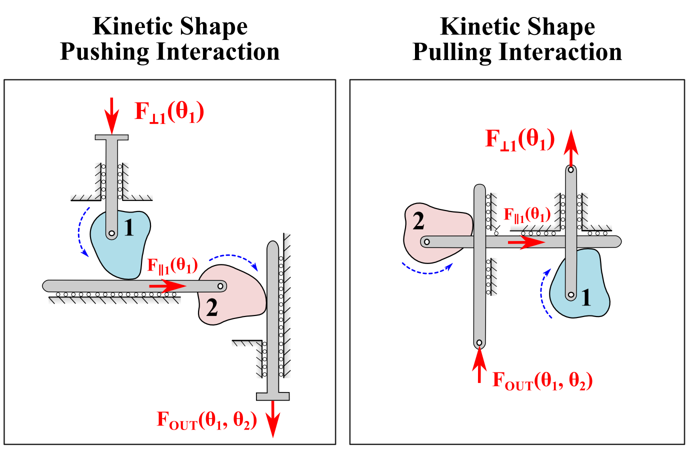

FIGURE 2. Pushing and pulling force transfer configuration of kinetic shapes. Physically, a pushing configuration may introduce imbalance and misalignment in the system.

KINETIC SHAPE SYSTEM FORCE OPERATIONS

The concept of the kinetic shape system (KSS) can be extrapolated to a greater number of KSs involving more intricate KS arrangements to produce a wide range of position-dependent force profile outputs. KSSs can be arranged to produce computational operations with physical forces, such as arithmetic and conditional operations, as described below.

Force Division

This KSS arrangement is set up to where the incoming force onto the system is divided by some predefined position-dependent factor. While one single KS divides the incoming force by a factor of $\frac{F_{\parallel}(\theta)}{F_{\perp}(\theta)}$, it is not by definition a KSS. Connecting multiple KSs in series as shown in Figure 3 *(Top Left)* will result in multiple force divisions. The final KSS output force profile is broadly defined by Equation 4.

$$F_{OUT}(\theta_1, \theta_2, ...\theta_n) = F_{\perp 1}(\theta_1)\frac{F_{\parallel 1}(\theta_1)}{F_{\perp 1}(\theta_1)}\frac{F_{\parallel 2}(\theta_2)}{F_{\parallel 1}(\theta_1)} \cdots \cdots \frac{F_{\parallel n}(\theta_n)}{F_{\parallel n-1}(\theta_{n-1})} \tag{4}$$

Note that the concluding system force profile, F_{OUT}, is dependent on the rotational position of all the KSs involved.

Force Multiplication

Force multiplication is the exact opposite of force division. That is, the "ground" or the platform on which the KS rolls over applies a force at the ground contact point. This reverse force causes a KS axle to push away and is perpendicular to the ground. Force multiplication is shown in Figure 3 *(Top Right)* while the concluding KSS output force profile is generally defined by Equation 5.

$$F_{OUT}(\theta_1, \theta_2, ...\theta_n) = F_{\parallel 1}(\theta_1)\frac{F_{\parallel 1}(\theta_1)}{F_{\perp 1}(\theta_1)}\frac{F_{\perp 1}(\theta_1)}{F_{\perp 2}(\theta_2)} \cdots \cdots \frac{F_{\perp n-1}(\theta_{n-1})}{F_{\perp n}(\theta_n)} \tag{5}$$

Practically this method requires a high amount of friction/adherence between at ground surface contact.

Force Addition and Subtraction

Forces can be added by aligning the shape's reaction forces ($F_{\parallel 1} + F_{\parallel 2}$) to push onto a shared platform. Redefinition or reorientation of the KSs can result in subtraction of reaction forces $(F_{\|1} - F_{\|2})$. While the schematics for addition and subtraction in Figure 3 show two involved KSs, addition and

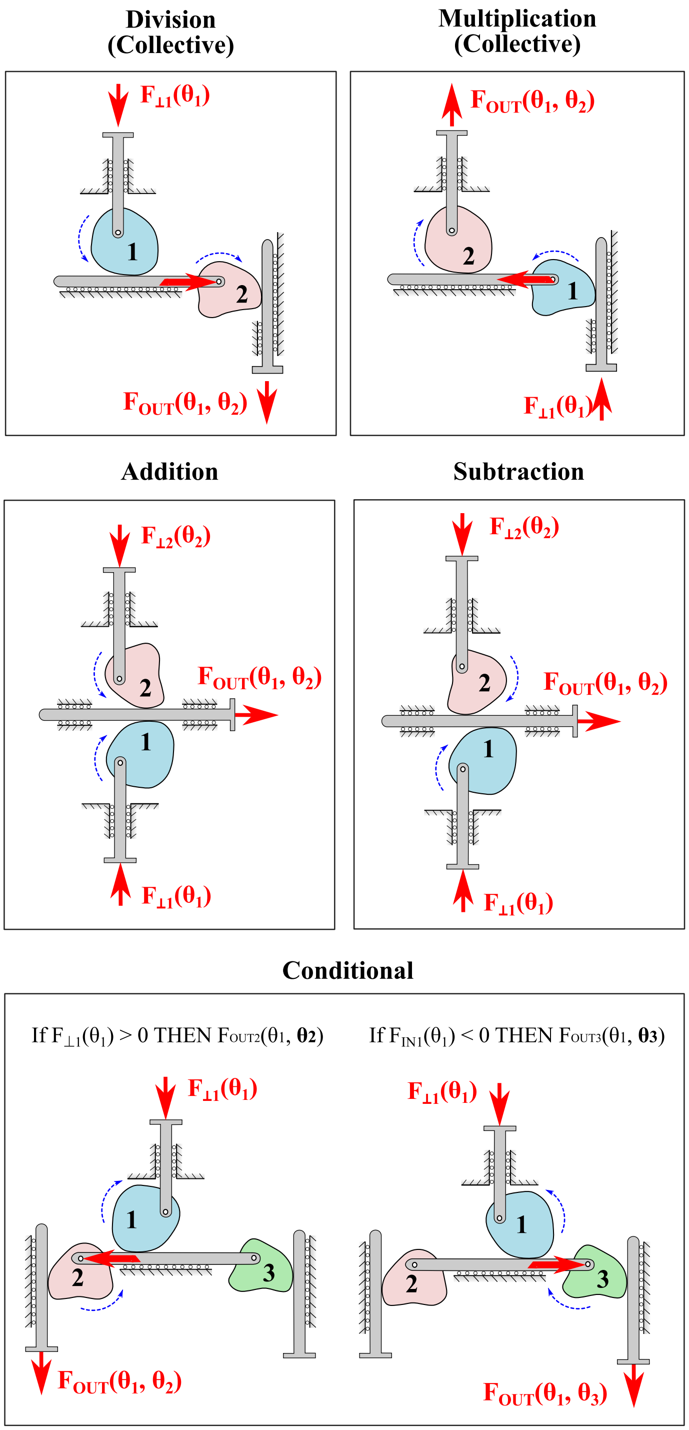

FIGURE 3. Basic kinetic shape system force operations. Manipulating kinetic shape interactions can result in arithmetic and conditional operations using physical forces.

subtraction of forces can be achieved with only one KS. For example, one KS may push on one side of a platform, while a force is applied to the opposite side of the platform unrelated to a KS. KSS addition of forces is described by Equation 6, while KSS subtraction is defined by Equation 7.

$$\begin{aligned} F_{OUT}(\theta_1, \theta_2, \ldots \theta_n) &= \\ & F_{\perp 1}(\theta_1)\frac{F_{\|1}(\theta_1)}{F_{\perp 1}(\theta_1)} + F_{\perp 2}(\theta_2)\frac{F_{\|2}(\theta_2)}{F_{\perp 2}(\theta_2)} + \cdots \\ & \cdots + F_{\perp n}(\theta_n)\frac{F_{\|n}(\theta_n)}{F_{\perp n}(\theta_n)} \end{aligned} \tag{6}$$

$$\begin{aligned} F_{OUT}(\theta_1, \theta_2, \ldots \theta_n) &= \\ & F_{\perp 1}(\theta_1)\frac{F_{\|1}(\theta_1)}{F_{\perp 1}(\theta_1)} - F_{\perp 2}(\theta_2)\frac{F_{\|2}(\theta_2)}{F_{\perp 2}(\theta_2)} - \cdots \\ & \cdots - F_{\perp n}(\theta_n)\frac{F_{\|n}(\theta_n)}{F_{\perp n}(\theta_n)} \end{aligned} \tag{7}$$

Force Dependent Conditional Operations

Kinetic shape systems can be used to make decisions that depend on position-dependent mechanical force inputs. For example, as a mechanical device component applies a force onto a KSS, it may be desirable to output two different types of force profiles depending on the orientation of the mechanical device component which applied the initial force. In essence, a KSS can be arranged as a kinetic conditional statement operator depending on the reaction force profile of the conditional KS. Equation 8 shows the KSS conditional statement of Figure 3.

$$\begin{aligned} &\text{IF } F_{\perp 1}(\theta_1) > 0 \text{ THEN} \\ &\quad F_{OUT}(\theta_1, \theta_2) = F_{\perp 1}(\theta_1)\frac{F_{\|1}(\theta_1)}{F_{\perp 1}(\theta_1)}\frac{F_{\|2}(\theta_2)}{F_{\|1}(\theta_1)} \\ &\text{IF } F_{\perp 1}(\theta_1) < 0 \text{ THEN} \\ &\quad F_{OUT}(\theta_1, \theta_3) = F_{\perp 1}(\theta_1)\frac{F_{\|1}(\theta_1)}{F_{\perp 1}(\theta_1)}\frac{F_{\|3}(\theta_3)}{F_{\|1}(\theta_1)} \end{aligned} \tag{8}$$

Other Force Operations

Other KSS operations and included components are possible to further control the final force output of a KSS. For example, a spring can be included into the design of a KSS which may assist or resist internal KSS forces (i.e., $F_n(\theta) - F_k$). These supplemental forces can be simply and appropriately added to the applied or reacted forces of each KS.

Non-Conservative System Losses

No mechanical force transfer can be designed without a finite amount of non-conservative system losses. Depending on the quality of a physical KSS, kinetic losses due to striction, friction, or misalignment may occur. These kinetic losses in the

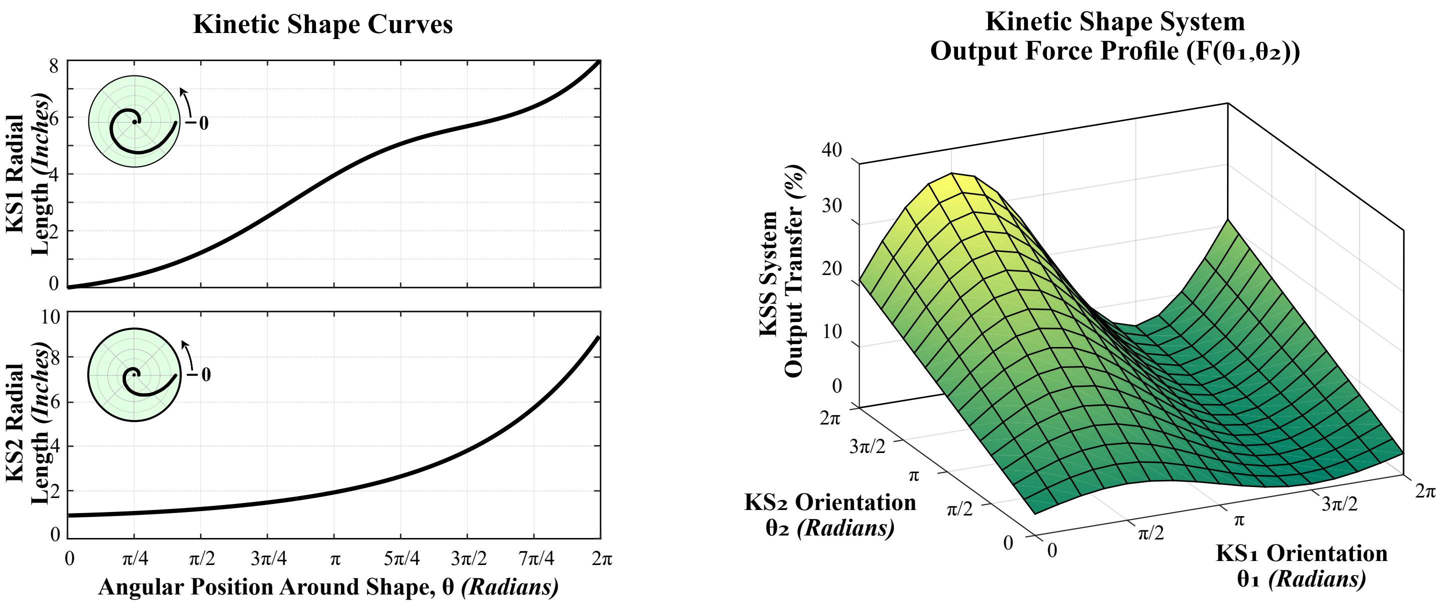

FIGURE 4. ***(Left)*** Physical curves of kinetic shapes used in the second degree kinetic shape system experimental setup. The kinetic shapes are defined independently. ***(Right)*** Force profile resulting from the interaction (force division) of the two kinetic shapes.

KSS are accounted for by including a force transfer coefficient, represented by the variable D_f. The *force transfer coefficient, D_f,* is the percentage of the force *saved*, or successfully transfered, during transfer due to imperfections in the physical design. The force transfer coefficient has a range from zero to one, where zero indicates no force transfer and one indicates no force losses. To apply the force transfer coefficient to a KSS force transfer step, the coefficient D_f is simply multiplied for each step. For example, if one is to add a force transfer coefficient to each step in a chain of multiplication operations, Equation 5 would turn into Equation 9.

$$F_{OUT}(\theta_1, \theta_2, ...\theta_n) = F_{IN1}(\theta_1)\mathbf{D_{f1}}\frac{F_{IN1}(\theta_1)}{F_1(\theta_1)}\mathbf{D_{f2}}\frac{F_1(\theta_1)}{F_2(\theta_2)}\cdots \cdots \mathbf{D_{fn}}\frac{F_{n-1}(\theta_{n-1})}{F_n(\theta_n)} \tag{9}$$

If an entire KSS has one force input and one force output, it is also possible to specify the cumulative force decay coefficient for the entire KSS from input to output. Note that for a multiplication and division operation, the cumulative force decay coefficient multiplies at each step, while in addition and subtraction it does not.

KINETIC SHAPE SYSTEM EXAMPLE SIMULATION

To further communicate the essence of a KSS, we will generate and analyze a virtual KSS. The KSS that will be examined is a second degree KSS with two force devision operations. A system input force is applied to the first KS (noted as subscripts 1), the first KS reacts with an position-dependent output, which in turn is applied to the second KS (noted as subscript 2). The second KS reacts to produce the system output force.

First the curve of the two interacting KSs are defined by specifying the forces applied to the shapes and forces with which the shapes react. We are able to alternatively define these forces as a percentage, which does not change the resulting force operation. It is the percentage of force transfer based on the force input to the system.

To construct each KSs in the KSS, we use the original KS Equation 1. The applied force ($F_{\perp 1}$) to the first KS is defined as Equation 10, while the force with which the KS reacts ($F_{\parallel 1}$) can be defined as Equation 11.

$$F_{\perp 1}(\theta_1) = 100 \quad [\%] \tag{10}$$

$$F_{\parallel 1}(\theta_1) = 25\,sin(\theta_1) + 35 \quad [\%] \tag{11}$$

Note that the force profile output of the first KS ($F_{\parallel 1}$) varies sinusoidally. These force definitions for the first KS will result in its shape curve, $R_1(\theta)$, defined by Equation 12 and shown in Figure 4 (Left Top).

$$R_1(\theta_1) = 1.0\,exp\left[\frac{7\theta_1}{20}\;\frac{cos(\theta_1)}{4}\right]_{\theta_1=0}^{\theta_1=2\pi} \quad [inches] \tag{12}$$

Likewise the second KS is defined by its rotational position-dependent force profiles in Equations 13 and 14

$$F_{\perp 2}(\theta_2) \;=\; 100 \quad [\%] \tag{13}$$

$$F_{\parallel 2}(\theta_2) \;=\; \frac{50}{2\pi}(\theta_2)+10 \quad [\%] \tag{14}$$

For this KS a linearly increasing force profile output was chosen. The second KSs curve is defined by Equation 15 and shown in Figure 4 (Left Bottom).

$$R_2(\theta_2) \;=\; 1.0\, exp\left[\frac{\theta_2^2}{8\pi}\;\frac{\theta_2}{10}\right]_{\theta_2=0}^{\theta_2=2\pi} \quad [inches] \tag{15}$$

Using Equations 10, 11, 13, and 14, and inserting them into Equations 4, the cumulative KSS output is defined as Equation 16.

$$F_{out}(\theta_1,\theta_2) = F_{\perp 1}(\theta_1)\left[\frac{F_{\parallel 1}(\theta_1)}{F_{\perp 1}(\theta_1)}\right]\left[\frac{F_{\parallel 2}(\theta_2)}{F_{\perp 2}(\theta_2)}\right] \tag{16}$$

Results

This resulting force profile can be seen in Figure 4 (*Right*). Note that because the final force profile is dependent on the rotational position of both KSs in the system, the system output force profile is shown as a surface with x and y axes representing the inputs and the z (height) representing the force transfer. This simulated KSS is able to produce any point on this force profile output surface. Note that a KSS degree higher than two may be harder to visually communicate, but is theoretically possible.

CONCLUSIONS

In this paper we have introduced the concept of kinetic shape systems (KSSs), which are a collection of position-dependent kinetic shapes (KSs) collectively and mechanically processing forces inputted into the KSS, and outputting a predictable force profile. We have presented ways to arrange KSSs in order to produce arithmetic and conditional computational operations using mechanical forces. A virtual simulation of a KSS has shown the theoretical validity of a second degree force devision KSS system. While this paper is a mere introduction to the broad possibilities of KSSs, it leaves for further research including the physical verification of this theoretical concept. Furthermore, the authors future intentions are to develop a clear KSS notation system in order to effectively communicate the various arrangements of KSs in a KSS.

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